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ALFA CODE DISPLAYED

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ABSTRACT.

Recently developed at the University of Wisconsin-Madison, the global full-wave code, ALFA, simulating RF plasma heating and current drive in tokamak plasmas, is presented. This code solves Maxwell's equations in a toroidal tokamak plasma as a boundary problem using the finite element Galerkin scheme. The ALFA code includes many features of the global full-wave code, FASTWA [1-3] developed also at the University of Wisconsin-Madison, but significantly enhances it in a few directions. Among additional features of the ALFA code are the arbitrary toroidal geometry of a plasma and a vacuum vessel of a tokamak, ability to treat Alfven frequencies (below the ion cyclotron frequency), obtain solution for both, fast and slow waves simultaneously, including mode conversion between them, incorporate large Larmor radius effects, and treat high cyclotron harmonic absorption. This paper discusses the physical and numerical basis of the code and displays a few examples of calculations for parameters of the TPX tokamak.

INTRODUCTION.

ICRF plasma heating via fast waves was confirmed to be an effective method in present tokamaks. For future thermonuclear reactors, however, many questions of wave penetration, absorption and current drive still have to be clarified. Many factors, such as directionality of wave excitation and penetration in the plasma, dependence of wave absorption and current drive efficiency on plasma and magnetic field geometry, effective RF power deposition into electrons and parasitic absorption of wave energy by ions might play a very significant role and have to be analyzed more vigorously. At the same time the ray-tracing methods, normally used for the high-frequency wave examination, cannot be generally utilized for Alfven or ICRF waves because of effects of diffraction, wave interference and mode conversion can be significant. For these relatively low frequencies the full-wave approach, suggesting the direct solution of Maxwell equations in a plasma, is more appropriate. However, full-wave calculations require significant amount of computer time, especially when carried out in more then one dimension. Various simplifications and approximations are important to make a solution feasible. The ALFA code is essentially a 2D full-wave code, but it obtains a 3D picture of RF wave fields and absorbed power via Fourier composition of solutions for many independent toroidal modes.

PHYSICAL AND COMPUTATIONAL BASIS.

The starting point for the ALFA code is a weak form of the Maxwell's equations [4, 5]:

$$\int dV \left\{ (\nabla \times \mathbf{E}^*)(\nabla \times \mathbf{E}) - \frac{\omega^2}{c^2} \mathbf{E}^* \overleftrightarrow{\varepsilon} \mathbf{E} \right\} = \frac{4\pi i \omega}{c^2} \int dV \ \mathbf{E}^* \cdot \mathbf{J_a} \ . \qquad (1)$$

where $\mathbf{E}$ is the wave electric field, $\overleftrightarrow{\varepsilon}$ is a hot plasma dielectric tensor, $\mathbf{J_a}$ is an antenna current, ω is a frequency, and c is the light speed. Integration is carried out over the volume inside the vacuum vessel of a tokamak. The general toroidal geometry was represented by a metric tensor

$$g_{ij} = \begin{pmatrix} h_\rho^2 & g & 0 \\ g & h_\vartheta^2 & 0 \\ 0 & 0 & R^2 \end{pmatrix} \qquad (2)$$

where R is a major radius at a given point with coordinates $x = x(\rho,\vartheta)$, $y = y(\rho,\vartheta)$, and

$$g = \frac{\partial x}{\partial \rho}\frac{\partial x}{\partial \vartheta} + \frac{\partial y}{\partial \rho}\frac{\partial y}{\partial \vartheta}, \quad h_\rho^2 = \left(\frac{\partial x}{\partial \rho}\right)^2 + \left(\frac{\partial y}{\partial \rho}\right)^2, \quad h_\vartheta^2 = \left(\frac{\partial x}{\partial \vartheta}\right)^2 + \left(\frac{\partial y}{\partial \vartheta}\right)^2 \qquad (3)$$

The tokamak magnetic field, $\mathbf{B}$, has toroidal and poloidal components:

$$\mathbf{B} = \mathbf{B_p} + \mathbf{B_\varphi} = B_0 R_0 [\, \nabla\varphi + f_1(\rho)\, \nabla\varphi \times \nabla\rho \,] \ , \qquad (4)$$

where φ is a toroidal angle, while B_0 and R_0 are magnetic field and the major radius at the magnetic axis. Function, $f_1(\rho)$, is proportional to the toroidal current.

The hot-plasma dielectric tensor, $\overleftrightarrow{\varepsilon}$, is well-known in coordinates related to the local magnetic field, and was translated through the metric coefficients to the toroidal coordinate system. It included terms for fundamental and general harmonic ion cyclotron absorption, and for electron absorption via TTMP, Landau and cross-term effect, through the proper corrections to the dielectric tensor components. The following terms in Eq. (1) have been used for ion harmonic absorption

$$(\mathbf{E}^* \overleftrightarrow{\varepsilon} \mathbf{E})_n = (\partial_+ \cdot E^+)^* \sum_i \frac{\omega_{pi}^2}{2\omega^2} \zeta_{oi} \rho_i^2 \frac{n^2}{\lambda_i^2} \cdot I_n(\lambda_i) e^{-\lambda_i} \cdot Z(\zeta_{ni}) (\partial_+ \cdot E^+) \ , \qquad (5)$$

where the standard notations are used: $\zeta_{ni} = (\omega - n\Omega_i)/k_\parallel v_i$, $\rho_i = v_i/\Omega_i$, $\lambda_i = 0.5 k_\perp^2 \rho_i^2$. I_n is the modified Bessel function, and Z is a plasma dispersion function of Fried and Conte. The E^+ electric field component and operator, ∂_+, are defined in the coordinates (ρ, η) perpendicular to the tokamak magnetic field

$$E^+ = E_\rho + iE_\eta, \qquad \partial_+ = \partial/\partial\rho + i\,\partial/\partial\eta \ . \qquad (6)$$

Computation of electron absorption terms needs expression for the parallel component of electric field, $E_\parallel$. We have used the one proposed in [1]:

$$E_\parallel = -i\, n_\parallel [\, \gamma B_\parallel - \nabla \cdot \mathbf{E}_\perp / k_0 \varepsilon_3 \,] \ , \qquad (7)$$

where $k_0 = \omega / c$, $n_\parallel = k_\parallel / k_0$, $\gamma = \omega v_e^2 / 2\Omega_e c^2$, and $\varepsilon_3 = 1 - (\omega_{pe}^2 / k_\parallel^2 v_e^2)\, Z'(\zeta_{oe})$. Then electron absorption for Landau damping, TTMP, and cross-term effect are given respectively by

$$P_{ld} = \frac{1}{\gamma n_{\parallel}^2} G\,|E_{\parallel}|^2, \quad P_{mp} = 2\gamma G\,|B_{\parallel}|^2, \quad P_{cr} = \frac{2}{n_{\parallel}} G\,\mathrm{Im}(B_{\parallel}^* E_{\parallel}^*) \tag{8}$$

with

$$G = \frac{\omega_{pe}^2}{8\pi\Omega_e} \zeta_{oe}\,\mathrm{Im}\,Z(\zeta_{oe}) \,. \tag{9}$$

To use $k_{\parallel}$ in above expressions we Fourier decomposed wave field components into poloidal and toroidal modes with n_{φ} and m being toroidal and poloidal wave numbers. Then, $k_{\parallel}$ is given by

$$k_{\parallel} = n_{\varphi} \cos\Theta / R + m \sin\Theta / h_{\vartheta} \,, \tag{10}$$

where Θ is an angle between the direction of the tokamak magnetic field and the toroidal direction. All terms in Eq. (1), and also terms for power deposition and for field components have been then written in terms of the poloidal and toroidal modes. As an example, the expression (7) for $E_{\parallel}$ becomes

$$E_{\parallel}(\rho,\vartheta) = -\frac{i\gamma(\rho,\vartheta)}{k_0} \sum_m k_{\parallel}^m(\rho,\vartheta)\, B_{\parallel}^m e^{im\vartheta} + \frac{i}{k_0^2} \sum_m \frac{k_{\parallel}^m(\rho,\vartheta)}{\varepsilon_3^m(\rho,\vartheta)} \nabla\cdot E_{\perp}^m(\rho)\, e^{im\vartheta} \,, \tag{11}$$

where index, m, denotes the poloidal mode. Using the traditional Galerkin scheme with the cubic finite elements led us to the linear algebraic system with the three-block diagonal matrix, which was solved via the standard mathematical subroutines available in FORTRAN.

NUMERICAL RESULTS.

The ALFA code, although presently in progress, has extended diagnostics with the possible output of up to 300 various plots during the single run of the code. It includes the dispersion relation analysis, 3D distribution of all wave electric and magnetic field components and power absorbed via various absorption mechanisms and mode conversion, and results on RF driven current. The code has been used recently for ICRF heating and current analysis in TPX [6], for Alfven wave current drive in Phaedrus-T [7], and for low frequency fast wave current drive in ITER [7].

Below we present an example of computations, via the ALFA code, of fast and slow (IBW) waves for the standard minority heating regime (deuterium plasma with 5% of hydrogen) in TPX at frequency, f = 60 MHz, central density, $n_e(0) = 10^{20}\ m^{-3}$, magnetic field, B_0 = 1T, and temperature, $T_e(0) = T_i(0)$ = 10 keV. One toroidal mode, n_{φ} = 15, and 15 different poloidal modes are taken into analysis for this example. Fig. 1 gives the contour lines of the fast wave dispersion root, $k_{\perp}^2$. Fast change of $k_{\perp}^2$ corresponds to the mode conversion region. Fig. 2 represents the distribution of $\mathrm{Re}(E^+)$ in the poloidal cross-section. Fast wave launched by the antenna and mode converted IBW are seen clearly. The following, Figs. 3-6, show the power deposition profiles, respectively, via the D-ions (19.33%), H-ions (65.64%), electrons (14.13%), and deposited by IBW (0.9%).

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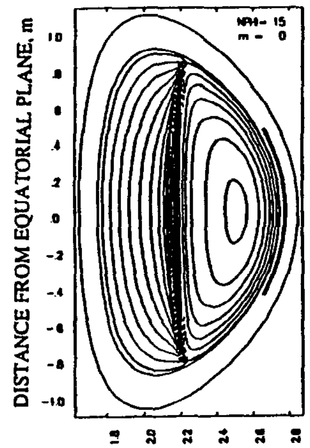


Fig. 1

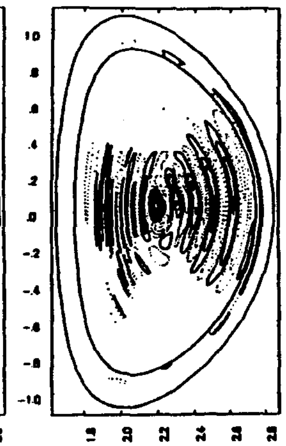


Fig. 2

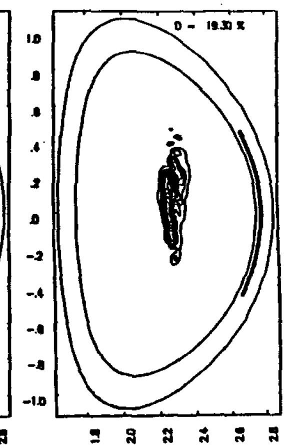
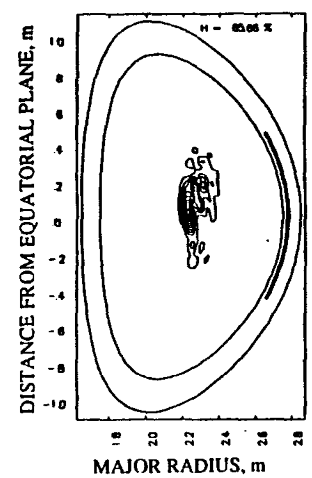


Fig. 3



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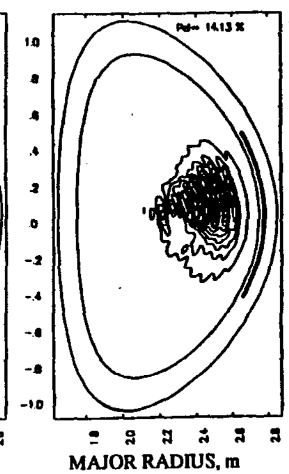

Fig. 5

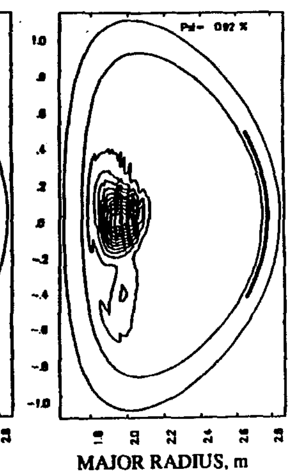

Fig. 6